

Exercise 18

(a) Find the differential dy and (b) evaluate dy for the given values of x and dx .

$$y = \frac{x+1}{x-1}, \quad x = 2, \quad dx = 0.05$$

Solution

Compute the derivative of y .

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{x+1}{x-1} \right) \\ &= \frac{\left[\frac{d}{dx}(x+1) \right] (x-1) - \left[\frac{d}{dx}(x-1) \right] (x+1)}{(x-1)^2} \\ &= \frac{(1)(x-1) - (1)(x+1)}{(x-1)^2} \\ &= \frac{-2}{(x-1)^2} \end{aligned}$$

Consequently, the differential of $y = (x+1)/(x-1)$ is

$$dy = \frac{-2}{(x-1)^2} dx.$$

If $x = 2$ and $dx = 0.05$, then

$$dy = \frac{-2}{(2-1)^2} (0.05) = -\frac{1}{10} = -0.1.$$